Maths A-Level Summer Work

Westcliff High School
for Boys

WHSB Mathematics Department

Introduction

The Mathematics Department at WHSB wants to ensure that you make the best start to your Year 12 study as possible. You may have heard about a **step up** from GCSE to A Level, and that even Grade 9 students at GCSE level can find the pace and content of the course challenging, especially in the first few weeks. Therefore, we insist on you completing the work in this booklet before you begin the course in September. All of these topics are in the GCSE syllabus, although if you are an external applicant to WHSB you may not have been taught them as not all schools teach the entire higher syllabus to all pupils. We have provided worked examples and instruction throughout so that if any of these topics are not familiar, you should be able to teach yourself from this booklet. These topics essentially cover the first four chapters of Pure Mathematics 1 (a few lessons covering any new content will be taught in the first week).

In the first week of the course you will take a test on the contents of this booklet.

If you score below 70% you will be required to complete additional work and take a second test around two weeks later. If you score below 70% on this second test, it is our view that you are unlikely to be able to successfully complete the A level course and we would recommend you discontinue your study of Mathematics in order to pursue a subject for which you are better suited.

Contents

Chapter 1

Outline of the Course

Exam Board Specification: Edexcel 8371 / 9371

Why Study Mathematics?: Higher level Mathematics is becoming more beneficial in a world that is technologically dependent. Alongside teaching students the necessary building blocks for many other subjects such as Physics, Engineering and Economics, we also aim to develop the students' ability to think and tackle problems in a logical and systematic manner. These thinking and study skills will produce highly effective learners in all subjects, not just great Mathematicians.

1.1 The A level Course

<table>
<tr><td colspan="4"></td></tr>
<tr><td>Unit 1</td><td>Pure Mathematics 1</td><td>120 minutes examination</td><td>$33\frac{1}{3}\%$</td></tr>
<tr><td colspan="4">This unit includes much of what has been studied at GCSE, extending this to set a firm foundation for the Pure Mathematics done throughout A level. Students will be learning: Proof; Algebra and Functions; Coordinate Geometry in the $(x,\ y)$ plane; Sequences and Series; Trigonometry; Exponentials and Logarithms; Differentiation; Integration and Vectors.</td></tr>
<tr><td>Unit 2</td><td>Pure Mathematics 2</td><td>120 minutes examination</td><td>$33\frac{1}{3}\%$</td></tr>
<tr><td colspan="4">This unit expands on the work done in Pure Mathematics 1, taking the topics learned previously and expanding upon them. Students will be learning: Proof; Algebra and Functions; Coordinate Geometry in the (x, y) plane; Sequences and Series; Trigonometry; Differentiation; Integration and Numerical Methods.</td></tr>
<tr><td>Unit 3</td><td>Statistics and Mechanics</td><td>120 minutes examination</td><td>$33\frac{1}{3}\%$</td></tr>
<tr><td colspan="4">These topics are very similar to the Statistics 1 and Mechanics 1 courses from the old specification. In Statistics students will be learning about Statistical sampling, Data presentation and Interpretation, Probability, Statistical Distributions and Statistical Hypothesis testing. In Mechanics students will be learning about Quantities and units in Mechanics, Kinematics, Forces and Newton's laws and Moments. This unit is split in to two sections (Statistics and then Mechanics) and half the marks are awarded for each section.</td></tr>
</table>

Chapter 2

Reading list

There are many popular books relating to Mathematics published every year and, considering your intention to study Mathematics at Advanced Level, these may well be of interest to you. Certainly if you choose to study Mathematics at University you will need to demonstrate your interest in the subject beyond the curriculum and one way of doing so is by your wider reading. You may wish to read the following, but there are of course many other books which you can find in the school library or purchase yourself.

David Acheson, **1089 and All That** (Oxford: OUP, 2002)

E. T. Bell, **Men of Mathematics** (Touchstone Books, 1937)

John Derbyshire, **Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics** (Plume Books, 2004)

William Dunham, **The Mathematical Universe** (Wiley, 1994); **Journey Through Genius** (Penguin Books, 1991)

Darrell Huff, **How to Lie with Statistics** (London: Penguin, 1991)

Marcus du Sautoy, **Music of the Primes** (Harper Perennial, 2004)

Simon Singh, **The Code Book: The Secret History of Codes and Code-breaking** (London: Fourth Estate, 1999); **Fermat's Last Theorem** (Delta, 1997); **The Simpsons and Their Mathematical Secrets** (Bloomsbury 2014)

Ian Stewart, **Why Beauty is Truth: The History of Symmetry** (Basic Books, 2008);

Professor Stewart's Casebook of Mathematical Mysteries (Profile Books, 2015)

Chapter 3

What you should already know and where to find it

The notes that follow are designed to remind you about material studied at GCSE that will appear in the first few chapters of Pure Mathematics 1.

If you need additional support with any of these materials the Dr Frost, Maths Genie or Physics and Maths Tutor websites are very useful.

This booklet covers most of the topics listed here so work through all of the preparatory material provided before seeking additional help. When you begin the course in September you will also have access to Microsoft Teams, where lots of materials will be placed. To access this you will need to use your school email.

There is some new material in the first four chapters of Pure Mathematics 1 and this will be formally taught in class in the first week, before the Chapters 1-4 test:

Section 1.8: Rationalising the denominator of a surd

Section 2.6: The discriminant

Section 3.4: Simultaneous linear inequalities

Section 3.5: Quadratic inequalities

Section 4.4: Intersection of graphs

Section 4.5: Graph transformations

The below topics are based on the Pure Mathematics 1 textbook list. Material for these topics is easy to find on websites like www.physicsandmathstutor.com. The material you should know is:

Chapter 1: Algebra and Functions
Section 1.1: Simplifying an expression by collecting like terms *Where did we do this?* This is Key Stage 3-level Algebra *Where can I get extra practice?* If you need help with this then this course is not for you.
Section 1.2: The laws of indices *Where did we do this?* This is Key Stage 3-level algebra *Where can I get extra practice?* If you need help with this then this course is not for you!
Section 1.3: Expanding an expression *Where did we do this?* This is Key Stage 3-level algebra *Where can I get extra practice?* If you need help with this then this course is not for you!
Section 1.4: Factorising an expression *Where did we do this?* This is Key Stage 3-level algebra *Where can I get extra practice?* If you need help with this then this course is not for you!
Section 1.5: Factorising a quadratic expression ***Video Help*** *https://www.youtube.com/watch?v=6NIdTWcpK1s&list=PLhfTFUpngHaW6s54XUUZmHJvC57KyT316&index=3* *Where can I get extra practice?* https://www.mathsgenie.co.uk/factorising-harder.html
Section 1.6: The laws of indices for all rational exponents ***Video Help*** https://www.youtube.com/watch?v=pUekEPvXWCU&list=PLhfTFUpngHaW6s54XUUZmHJvC57KyT316&index=1 *Where can I get extra practice?* https://www.mathsgenie.co.uk/resources/4-indices.pdf https://www.mathsgenie.co.uk/resources/6-fractional-and-negative-indices.pdf https://www.physicsandmathstutor.com/pdf-pages/?pdf=https%3A%2F%2Fpmt.physicsandmathstutor.com%2Fdownload%2FMaths%2FA-level%2FC1%2FTopic-Qs%2FOCR-MEI%2FC1%2520Algebra%2520-%2520Indices%25201%2520QP.pdf https://www.physicsandmathstutor.com/pdf-pages/?pdf=https%3A%2F%2Fpmt.physicsandmathstutor.com%2Fdownload%2FMaths%2FA-level%2FC1%2FTopic-Qs%2FOCR-MEI%2FC1%2520Algebra%2520-%2520Indices%25202%2520QP.pdf
Section 1.7: The use and manipulation of surds ***Video Help*** https://www.youtube.com/watch?v=D6tGkiAGq2E&list=PLhfTFUpngHaW6s54XUUZmHJvC57KyT316&index=5 *Where can I get extra practice?* https://www.mathsgenie.co.uk/resources/7-surds.pdf

Section 1.8: Rationalising the denominator of a fraction when it is a surd
Video Help
https://www.youtube.com/watch?v=vKATQNlrv5o&list=PLhfTFUpngHaW6s54XUUZmHJvC57KyT316&index=6
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/as-pure-algebraic-expressions.pdf
https://www.physicsandmathstutor.com/pdf-pages/?pdf=https%3A%2F%2Fpmt.physicsandmathstutor.com%2Fdownload%2FMaths%2FA-level%2FC1%2FTopic-Qs%2FOCR-MEI%2FC1%2520Algebra%2520-%2520Surds%2520QP.pdf

Chapter 2: Quadratic Functions

Section 2.1: Plotting the graph of quadratic functions
Where did we do this? This is Key Stage 3-level algebra
Where can I get extra practice? If you need help with this then this course is not for you!

Section 2.2: Solving quadratic equations by factorisation
Video Help
https://www.youtube.com/watch?v=UEYen7bjs0o&list=PLhfTFUpngHaVKopUpsRPsQRMBPiU2kpDQ&index=1
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/5-solving-quadratics-by-factorising.pdf

Section 2.3: Completing the square
Video Help
https://www.youtube.com/watch?v=AdWQwPqcIc8&list=PLhfTFUpngHaVKopUpsRPsQRMBPiU2kpDQ&index=2
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/9-completing-the-square.pdf

Section 2.4: Solving quadratic equations by completing the square
Video Help
https://www.youtube.com/watch?v=AdWQwPqcIc8&list=PLhfTFUpngHaVKopUpsRPsQRMBPiU2kpDQ&index=2
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/as-pure-completing-the-square.pdf
https://www.physicsandmathstutor.com/pdf-pages/?pdf=https%3A%2F%2Fpmt.physicsandmathstutor.com%2Fdownload%2FMaths%2FA-level%2FC1%2FTopic-Qs%2FOCR-Set-1%2FC1%2520Completing%2520the%2520Square.pdf

Section 2.5: Solving quadratic equations by using the formula
Video Help
https://www.youtube.com/watch?v=UEYen7bjs0o&list=PLhfTFUpngHaVKopUpsRPsQRMBPiU2kpDQ&index=1
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/7-quadratic-formula.pdf

Chapter 3: Equations and Inequalities

Section 3.1: Solving simultaneous linear equations by elimination
Video Help
https://www.youtube.com/watch?v=FvwkZQ0Ymk0&list=PLhfTFUpngHaWJ5wPMJo_1CU954NqthqcT&index=1
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/5-simultaneous-equations.pdf

Section 3.2: Solving simultaneous linear equations by substitution
Video Help
https://www.youtube.com/watch?v=FvwkZQ0Ymk0&list=PLhfTFUpngHaWJ5wPMJo_1CU954NqthqcT&index=1
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/5-simultaneous-equations.pdf

Section 3.3: Using substitution when one equation is linear and the other is quadratic
Video Help
https://www.youtube.com/watch?v=RIIqfS9rXQA&list=PLhfTFUpngHaWJ5wPMJo_1CU954NqthqcT&index=2
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/9-quadratic-simultaneous-equations.pdf
https://www.physicsandmathstutor.com/pdf-pages/?pdf=https%3A%2F%2Fpmt.physicsandmathstutor.com%2Fdownload%2FMaths%2FA-level%2FC1%2FTopic-Qs%2FOCR-Set-1%2FC1%2520Simultaneous%2520Equations.pdf

Chapter 4: Sketching curves

Section 4.2: Interpreting the graphs of cubic functions
Video Help
https://www.youtube.com/watch?v=slYwhWn16ko&list=PLhfTFUpngHaXt-XLcnpVQxMA320SqbpzY&index=1
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/as-pure-sketching-and-transforming-curves.pdf
https://www.physicsandmathstutor.com/pdf-pages/?pdf=https%3A%2F%2Fpmt.physicsandmathstutor.com%2Fdownload%2FMaths%2FA-level%2FC1%2FTopic-Qs%2FOCR-MEI%2FC1%2520Curve%2520Sketching%2520-%2520Factorising%2520%26%2520Sketching%2520Polynomials%25201%2520QP.pdf

Section 4.4: Using the intersection points of graphs of functions to solve equations
Video Help
https://www.youtube.com/watch?v=03gyABEr8Zo&list=PLhfTFUpngHaXt-XLcnpVQxMA320SqbpzY&index=4
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/as-pure-sketching-and-transforming-curves.pdf

Section 4.5: The effect of the transformations $f(x + a)$ and $f(x) + a$
Video Help
https://www.youtube.com/watch?v=Vd62T4zpGUQ&list=PLhfTFUpngHaXt-XLcnpVQxMA320SqbpzY&index=5
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/9-transforming-graphs.pdf

Section 4.6: The effect of the transformations $f(ax)$) **and** $af(x)$
Video Help
https://www.youtube.com/watch?v=1IPfSPVzKQg&list=PLhfTFUpngHaXt-XLcnpVQxMA320SqbpzY&index=6
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/9-transforming-graphs.pdf

Section 4.7: Performing transformations on the sketches of curves
Video Help
https://www.youtube.com/watch?v=sCflUfI_gXk&list=PLhfTFUpngHaXt-XLcnpVQxMA320SqbpzY&index=7
Where can I get extra practice?
https://www.mathsgenie.co.uk/resources/as-pure-sketching-and-transforming-curves.pdf

Chapter 4

Preparation Material for September

4.1 Basic Algebra

4.1.1 Indices

You should be able to manipulate algebraic expression involving indices using the following rules:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Example:

(a) $2r^5 \times 4r^{-2} = 2 \times 4 \times r^5 \times r^{-2}$
$= 8 \times r^{5-2}$
$= 8r^3$

(b) $(3x^3)^2 \div x^4 = 27x^6 \div x^4$
$= 27x^2$

Questions:

1. (a) $a^4 \times a^3$ (b) $x^5 \div x^2$ (c) $(b^4 \times b^3) \div b^5$

(d) $a^4 \div a^3$ (e) $x^4 \times x^5$ (f) $(x^4 \times x^5)^2$

(g) $(a^5 \div a^2) \times a$ (h) $(a^3)^2 \times (a^2)^3$ (i) $(x^2 \times x^3)^2 \div x^4$

(j) $(b^4 \div b^2)^3$ (g) $(b^4)^3 \div (b^2)^3$ (l) $[a^4 \times (a^2)^3] \div a^8$

(m) $\frac{x^7 \times x^2}{x^4}$ (n) $\frac{a^4 \times (a^2)^2}{a^8}$ (o) $\frac{x^5}{x^2 \times x^2}$

4.1.2 Expanding and combining like terms

You can expand an expression by multiplying each term inside the bracket by the term(s) outside. Like terms can be combined to simplify an expression.

Example

(a) $-3x(5 - 2x) \equiv -15x + 6x^2$

(b) $2x(5x + 3) - 4(x^2 - 2x) \equiv 10x^2 + 6x - 4x^2 + 8x$

$\equiv 6x^2 + 14x$

(c) $(x + 4)(2x - 1) \equiv x(2x - 1) + 4(2x - 1)$

$\equiv 2x^2 - x + 8x - 4$

$\equiv 2x^2 + 7x - 4$

Questions:

2. Multiply out and simplify

(a)	$x(x + 1)$	(b)	$2(2x + 1)$	(c)	$2x(x - 1)$
(d)	$4x(2 + x)$	(e)	$5x(3 - 2x)$	(f)	$x^2(1 + x)$
(g)	$(x + 1)(x + 2)$	(h)	$(x + 1)(x - 1)$	(i)	$(x + 2)(x - 1)$
(j)	$(x - 3)(x - 2)$	(k)	$(1 + a)(1 + 2a)$	(l)	$(x + y)(x - y)$
(m)	$(ax + b)(cx - d)$	(n)	$(x + 1)^2$		

3. Multiply out and simplify

(a)	$(3x - 2y) + (4x - y)$	(b)	$(p - m) + (m - 2p)$
(c)	$5(x - 2) + 3(4 - x)$	(d)	$(3a + 2b) - (a - b)$
(e)	$2(3m + n) - 3(m - 3n)$	(f)	$(x - y) - (y - z) - (z - x)$
(g)	$3a(b - c) + (3b - 2)a$	(h)	$m(m - n) - n(n - m)$
(i)	$x(y - z) + y(z - x) + z(x - y)$	(j)	$3(2y + 5z) - 4(2y - x)$

4.1.3 Factorising

You can factorise expressions if each term has a common factor.

An expression will be fully factorised if the terms inside the bracket do not have any common factors.

Example:

Factorise

(a) $x^2 + 2x$ (b) $3x^2 - 9x$ (c) $x^3 - x^2$

Solutions

(a) Here, as both terms are multiples of x, we can write

$$x^2 + 2x \equiv x(x + 2)$$

(b) In this case, both terms are multiples of x and 3, giving,

$$3x^2 - 9x \equiv 3x(x - 3)$$

(c) In this example, both terms are multiples x^2,

$$x^3 - x^2 \equiv x^2(x - 1)$$

Quadratic expressions can be factorised into two brackets.

For quadratics for the form $x^2 + bx + c$ (i.e where the coefficient of x^2 is 1) we simply need to think of two numbers which add up to b and multiply to c. If these numbers are w and v then the quadratic factorises to $(x + w)(x + v)$.

Example:

Factorise

(a) $x^2 + 6x + 8$ (b) $x^2 - 5x + 6$

Solutions

(a) Two numbers which add to 6 and multiply to 8 are **4 and 2**. Hence the quadratic factorises to

$$(x + 4)(x + 2)$$

(b) Two numbers which add to -5 and multiply to 6 are **-3 and -2**. Hence the quadratic factorises to

$$(x - 2)(x - 3)$$

This method works because of the way we expand quadratic brackets. Imagine we are given the quadratic $(x + m)(x + n)$. Using one of the variety of methods to expand this (we recommend the grid method) we would get:

$\times$	x	$+n$
x	x^2	nx
$+m$	mx	mn

$$\text{Hence } (x + m)(x + n) \equiv x^2 + nx + mx + mn$$
$$\equiv x^2 + (m+n)x + mn$$

This shows that our previous method for factorising will always work on quadratics of this given form.
If we wanted to factorise a quadratic of the form $ax^2 + bx + c$ then we must use a slightly altered method. Firstly, we find 2 numbers which add to b and multiply to $a \times c$. Call these numbers w and v again. The quadratic is then rewritten as $(ax + w)(ax + v)$.

Please note: This is not the full factorisation.

At this point we must cancel down the coefficients in the brackets in pair, as if they were the numerator and denominator of a fraction (i.e Divide $(ax + w)$ by the highest common factor of a and w, and then divide $(ax + v)$ by the highest common factor of a and v). The resulting brackets will be the factors of your quadratic equation.

Example:

Factorise

(a) $3x^2 - 5x + 2$ (b) $6x^2 + 7x + 2$

Solutions

(a) Two numbers which add to -5 and multiply to 6 are **-3 and -2**. Hence, we rewrite the expression as $(3x - 3)(3x - 2)$. We note that the first factor has a highest common factor of 3, so diving by that gives our final factorised form

$$(x - 1)(3x - 2)$$

(b) Two numbers which add to 7 and multiply to 12 are **3 and 4**. Hence, we rewrite the expression as $(6x + 3)(6x + 4)$. We note that the first factor has a highest common factor of 3 and the second factor has a highest common factor of 2, so dividing respectively by these gives out final factorised form

$$(2x + 1)(3x + 2)$$

Questions:

4. Factorise

(a)	$x^3 + x^2$	(b)	$2x^2 - x^3$	(c)	$4x^3 - 2x^2$
(d)	$8x^3 + 4x^2$	(e)	$16x^2 - 36x^3$	(f)	$4x^3 + 22x^2$
(g)	$16x^2 - 6x^3$	(h)	$14x^3 + 21x^2$	(i)	$28x^3 - 49x^2$

5. *(This question concerns the difference of two squares)*

(a) Expand $(x + 5)(x - 5)$

(b) Factorise $x^2 - 25$

(c) Factorise each of the following:

(i)	$x^2 - 49$	(ii)	$x^2 - 64$	(iii)	$x^2 - 100$
(iv)	$x^2 - a^2$	(v)	$x^2 - 4b^2$		

6. Factorise

(a)	$x^2 + 7x + 12$	(b)	$x^2 + 8x + 7$	(c)	$x^2 + 11x + 18$
(d)	$x^2 + 12x + 27$	(e)	$x^2 + 17x + 70$	(f)	$x^2 + 6x + 8$
(g)	$x^2 + 16x + 28$	(h)	$x^2 + 18x + 77$	(i)	$x^2 + 16x + 63$

7. Factorise

(a)	$x^2 + x - 2$	(b)	$x^2 + x - 20$	(c)	$x^2 - x - 12$
(d)	$x^2 - 13x + 36$	(e)	$x^2 - 10x + 16$	(f)	$x^2 + x - 42$
(g)	$x^2 + 13x - 30$	(h)	$x^2 - 17x + 72$	(i)	$2x^2 - 2x - 99$

8. Factorise

(a)	$2x^2 + 3x + 1$	(b)	$2 + 7p + 3p^2$	(c)	$2y^2 - 5y + 3$
(d)	$2 - m - m^2$	(e)	$3r^2 - 2r - 1$	(f)	$5 - 19y - 4y^2$
(g)	$4 - 13a + 3a^2$	(h)	$5x^2 - 8x - 4$	(i)	$4x^2 + 8x + 3$
(j)	$9s^2 - 6s + 1$	(k)	$4m^2 - 25$	(l)	$2 - y - 6y^2$
(m)	$4u^2 + 17u + 4$	(n)	$6p^2 + 5p - 4$	(o)	$8x^2 + 19x + 6$

4.1.4 Surds and rationalising the denominator

You can manipulate surds using the following rules:

$$\sqrt{ab} \equiv \sqrt{a} \times \sqrt{b} \qquad\qquad \sqrt{\frac{a}{b}} \equiv \frac{\sqrt{a}}{\sqrt{b}}$$

These follow the same rules as those of indices, namely $(a^m)^n \equiv a^{mn}$ because a surd is just an index (recall that $a^{\frac{n}{m}} \equiv \sqrt[m]{a^n}$.)

To rationalise the denominator of a fraction containing a surd of the form $\frac{a}{\sqrt{b}}$ you need to multiply by $\frac{\sqrt{b}}{\sqrt{b}}$ to obtain $\frac{a\sqrt{b}}{b}$.

Example:

Simplify:

(a) $\sqrt{12} \equiv \sqrt{4 \times 3} \equiv \sqrt{4} \times \sqrt{3} \equiv 2\sqrt{3}$

(b) $\sqrt{20} + \sqrt{5} \equiv \sqrt{4 \times 5} + \sqrt{5} \equiv \sqrt{4} \times \sqrt{5} + \sqrt{5} \equiv 2\sqrt{5} + \sqrt{5} \equiv 3\sqrt{5}$

(c) $\frac{\sqrt{72}-\sqrt{8}}{\sqrt{2}} \equiv \frac{\sqrt{36\times2}-\sqrt{4\times2}}{\sqrt{2}} \equiv \frac{6\sqrt{2}-2\sqrt{2}}{\sqrt{2}} \equiv \frac{4\sqrt{2}}{\sqrt{2}} \equiv 4$

(d) $\frac{\sqrt{99}}{\sqrt{11}} \equiv \sqrt{\frac{99}{11}} \equiv \sqrt{9} \equiv 3$

Questions:

9. Simplify

(a) $\sqrt{18}+\sqrt{50}$ (b) $\sqrt{48}-\sqrt{27}$ (c) $2\sqrt{8}+\sqrt{72}$

(e) $\sqrt{360}-2\sqrt{40}$ (e) $2\sqrt{5}-\sqrt{45}+3\sqrt{20}$ (d) $\sqrt{24}+\sqrt{150}-2\sqrt{96}$

10. Express in the form $a+b\sqrt{3}$

(a) $\sqrt{3}(2+\sqrt{3})$ (b) $4-\sqrt{3}-2(1-\sqrt{3})$ (c) $(1+\sqrt{3})(2+\sqrt{3})$

(d) $(4+\sqrt{3})(1+2\sqrt{3})$ (e) $(3\sqrt{3}-4)^2$ (f) $(3\sqrt{3}+1)(2-5\sqrt{3})$

11. Simplify

(a) $(\sqrt{5}+1)(2\sqrt{2}+3)$ (b) $(1-\sqrt{2})(4\sqrt{2}-3)$ (c) $(2\sqrt{7}+3)^2$

(d) $(3\sqrt{2}-1)(2\sqrt{2}+5)$ (e) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+2\sqrt{2})$ (f) $(3-\sqrt{8})(4+\sqrt{2})$

12. Express each of the following as simply as possible with a rational denominator

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{8}}$ (d) $\frac{14}{\sqrt{7}}$ (e) $\frac{3\sqrt{2}}{\sqrt{3}}$ (f) $\frac{\sqrt{5}}{\sqrt{15}}$

(g) $\frac{1}{3\sqrt{7}}$ (h) $\frac{12}{\sqrt{72}}$ (i) $\frac{1}{\sqrt{80}}$ (j) $\frac{3}{2\sqrt{54}}$ (k) $\frac{4\sqrt{20}}{3\sqrt{18}}$ (l) $\frac{3\sqrt{175}}{2\sqrt{27}}$

4.2 Quadratics

4.2.1 Graphs of Quadratic functions

You need to be able to sketch graphs of quadratic functions. You can do this by a drawing a table of values, for example you could take values of x between -4 and +4 and find the corresponding value of y by substituting it into the equation you are given, and then plotting the points on a graph.

If you are only sketching a graph, you need to know its shape and where it crosses the axes.

A quadratic graph is ∪ shaped if the coefficient of x^2 (a if the quadratic is given in the form $ax^2 + bx + c$) is positive.

A quadratic graph is ∩ shaped if the coefficient of x^2 (a if the quadratic is given in the form $ax^2 + bx + c$) is negative.

To find out where it crosses the y-axis you need to know what y is when $x = 0$, which is easy to find by substituting $x = 0$ into your equation.

To find out where it crosses the x-axis you need to know what x is when $y = 0$. You can do this by several methods, for example factorising, completing the square or using the quadratic formula. These methods are all covered below.

4.2.2 Solving Quadratics by factorising

You should be able to solve quadratics by factorising. If you have a factorised expression which is equal to 0, then you have two (or more) terms which multiply to give 0. For a set of numbers to multiply to 0 then one of those numbers must also be 0, and hence one of those factors is 0.

Example:

(a) Solve $x^2 - 5x + 6 = 0$

This factorises to $(x - 3)(x - 2) = 0$

so either $x - 3 = 0$ or $x - 2 = 0$.

Hence our solutions are $x = 3$ or $x = 2$.

(b) Solve $6x^2 - 7x + 2 = 0$

Firstly, we try and factorise this. We find two numbers that multiply to ac and add to b, so here multiply to 12 and add to −7. These numbers are −3 and −4.

Hence we rewrite the expression as $(6x-3)(6x-4)$ and we note that the first factor has a common factor of 3 and the second common factor 2.

Hence our factorised form is $(2x-1)(3x-2)$. Thus our original equation reduces to

$$(2x-1)(3x-2) = 0$$

so either $2x - 1 = 0$ or $3x - 2 = 0$.

Hence our solutions are $x = \frac{1}{2}$ or $x = \frac{2}{3}$

Questions:

13. Solve the following equations by factorization

(a) $x^2 + 2x - 35 = 0$ (b) $x^2 - 15x - 54 = 0$

(c) $x^2 - x - 90 = 0$ (d) $x^2 + 15x + 54 = 0$

(e) $x^2 + 20x + 51 = 0$ (f) $x^2 - 12x + 32 = 0$

(g) $x^2 - 24x + 143 = 0$ (h) $x^2 - 17x + 60 = 0$

(i) $x^2 - 14x - 176 = 0$ (j) $x^2 - 26x + 133 = 0$

(k) $x^2 + 7x - 44 = 0$ (l) $x^2 + 2x - 195 = 0$

(m) $2x^2 - 5x + 3 = 0$ (n) $2x^2 - 7x - 9 = 0$

(o) $2x^2 + 13x + 6 = 0$

14. Solve the following equations

(a) $x^2 - 16 = 0$ (b) $x^2 = 49$

(c) $4x^2 - 81 = 0$ (d) $9x^2 = 64$

These quadratics are known as the difference of two squares and you can factorise them quickly if you spot the pattern.

15. Solve the following equations

(a) $q^2 - 6q = -9$ (b) $x^2 + 81 = 18x$ (c) $y^2 = 22y - 121$

(d) $4(3x - 1) = 9x^2$ (e) $-25 = 4y(y - 5)$

16. Solve the following equations

(a) $x^2 = 25$ (b) $a^2 = 36$ (c) $y^2 = \frac{49}{4}$

(d) $b^2 - 16 = 0$ (e) $a^2 - 64 = 0$ (f) $x^2 - \frac{4}{81} = 0$

(g) $4y^2 = 0$ (h) $2x^2 = 32$ (i) $3p^2 - 27 = 0$

(j) $5p^2 - 20 = 0$ (k) $25b^2 - 40 = 9$ (l) $3b^2 - 8 = 4$

17. Find the solutions of each of the following equations

(a) $y^2 = y + 56$ (b) $12w^2 = 13w - 3$

(c) $11y = -4 - 6y^2$ (d) $c(c - 10) = 2$

(e) $q^2 = -2(q - 4)$ (f) $d(d + 2) = 3$

(g) $x(x - 5) = 84$ (h) $y(5y + 27) = 18$

(i) $3p^2 = 6p(2 + p)$ (j) $2x(4x + 5) = 3$

(k) $13x = 2(2x^2 + 5)$ (l) $2(10 - x^2) = 3x$

(m) $4y - 3 = 3y(y - 2)$ (n) $-12y - 9(y + 1) = 6y^2$

(o) $(a + 4)(a - 2) = -5$ (p) $(3x - 4)(x - 4) = -5$

4.2.3 Completing the square

Completing the square is a valuable technique for solving quadratics (and is how the quadratic equation is derived), but also for other topics such as graph transformations.

It is crucial that you are proficient at completing the square for AS Mathematics.

Completing the square is essentially a way of rewriting a quadratic expression in the form $a(x + b)^2 + c$, where a, b and c are constants. This makes it easy to solve if we have an equation, but also easy to see how the expression is a transformation of the graph $y = x^2$, which in turn allows us to find minimum or maximum points of the curve easily.

We'll start with the case where $a = 1$, so quadratic equations of the form $x^2 + bx + c$.

In order to find the completed square form, we have to first identify the **closest perfect square**. The closest perfect square is a perfect square (i.e a quadratic of the form $(x + a)^2$) where, when expanded, the coefficient of x is the same as in the equation we want to complete the square on.

Example

Given $x^2 + 8x + 10$ we would search for a perfect square of the form $(x + a)^2$ that, when expanded, gave $x^2 + 8x + c$, where c is a constant.

In this case, the closest perfect square is $(x + 4)^2$. When expanded, this gives $x^2 + 8x + 16$. *(Note that $(x + a)^2 \equiv x^2 + 2ax + a^2$, this will help you search for the correct closest perfect square).*

We then note what we need to subtract or add from the closest perfect square in order to obtain our original equation. $(x + 4)^2 \equiv x^2 + 8x + 16$ so $(x + 4)^2 - 6$ gives us $x^2 + 8x + 16 - 6$. Hence

$$x^2 + 8x + 10 \equiv (x + 4)^2 - 6$$

If $a \neq 1$ then you need to factorise first.

Given $2x^2 - 4x - 8$ we first factorise to $2(x^2 - 2x - 4)$. The coefficients here need not be nice numbers, so we suggest working in fraction form,

We then complete the square on $x^2 - 2x - 4$, which gives $(x - 1)^2 - 5$. Hence we get

$$2x^2 - 4x - 8 \equiv 2[(x - 1)^2 - 5].$$

The preferred form for the completed square form should not contain the square brackets above, so we expand the square brackets to give

$$2x^2 - 4x - 8 \equiv 2(x - 1)^2 - 10.$$

The reason we use completed square form is that we can solve quadratic equations from this point.

If $2x^2 - 4x - 8 = 0$

$2(x-1)^2 - 10 = 0$

$(x-1)^2 - 5 = 0$

$x - 1 = \sqrt{5}$ *Note the $\pm$ sign which is necessary when square rooting in equations*

then we rewrite this as $x = 1 \pm \sqrt{5}$

Questions:

18. Express in the form $(x + a)^2 + b$

(a) $x^2 + 2x + 4$ (b) $x^2 - 2x + 4$ (c) $x^2 - 4x + 1$ (d) $x^2 + 6x$

(e) $x^2 + 4x + 8$ (f) $x^2 - 8x - 5$ (g) $x^2 + 12x + 30$ (h) $x^2 - 10x + 25$

(i) $x^2 + 6x - 9$ (j) $18 - 4x + x^2$ (k) $x^2 + 3x + 3$ (l) $x^2 + x - 1$

(m) $x^2 - 18x + 100$ (n) $x^2 - x - \frac{1}{2}$ (o) $20 + 9x + x^2$ (p) $x^2 - 7x - 2$

(q) $5 - 3x + x^2$ (r) $x^2 - 11x + 37$ (s) $x^2 + \frac{2}{3}x + 1$ (t) $x^2 - \frac{1}{2}x - \frac{1}{4}$

19. Express in the form $a(x + b)^2 + c$

(a) $2x^2 + 4x + 3$ (b) $2x^2 - 8x - 7$ (c) $3 - 6x + 3x^2$ (d) $4x^2 + 24x + 11$

(e) $-x^2 - 2x - 5$ (f) $1 + 10x - x^2$ (g) $2x^2 + 2x - 1$ (h) $3x^2 - 9x + 5$

(i) $3x^2 - 24x + 48$ (j) $3x^2 - 15x$ (k) $70 + 40x + 5x^2$ (l) $2x^2 + 5x + 2$

(m) $4x^2 + 6x - 7$ (n) $-2x^2 + 4x - 1$ (o) $4 - 2x - 3x^2$ (p) $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$

20. Solve each equations by completing the square, giving your answers as simply as possible in terms or surds where appropriate.

(a) $y^2 - 4y + 2 = 0$ (b) $p^2 + 2p - 2 = 0$ (c) $x^2 - 6x + 4 = 0$

(d) $7 + 10r + r^2 = 0$ (e) $x^2 - 2x = 11$ (f) $a^2 - 12a - 18 = 0$

(g) $m^2 - 3m + 1 = 0$ (h) $9 - 7t + t^2 = 0$ (i) $u^2 + 7u = 44$

(j) $2y^2 - 4y + 1 = 0$ (k) $3p^2 + 18p = -23$ (l) $2x^2 + 12x = 9$

(m) $-m^2 + m + 1 = 0$ (n) $4x^2 + 49 = 28x$ (o) $1 - t - 3t^2 = 0$

4.2.3 Using the Quadratic formula

If you complete the square on the equation $ax^2 + bx + c = 0$ you get the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You need to know this equations and how to use it.

It is just a case of simply substituting in your values of *a*, *b* and *c* in to the given equation.

Example

$4x^2 - 3x - 2 = 0$ has values of $a = 4$, $b = -2$ and $c = -2$

so

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-2)}}{2(4)} = \frac{3 \pm \sqrt{41}}{8}$$

Hence your two answers are $x = \frac{3+\sqrt{41}}{8}$ and $x = \frac{3-\sqrt{41}}{8}$

4.2.4 Solving simultaneous equations by elimination and substitution

There are two main methods for solving simultaneous equations: elimination and substitution. Solving a pair of simultaneous equations is the same as finding where two lines intersect, i.e. you are finding the point at which their x and y values are the same for both equations, which in terms of coordinates means they both go through the same point and hence intersect.

Elimination involves manipulating the equations until one variable can be easily eliminated.This is the quicker method to solve them but only works for when we have linear terms in x and y, so **not** when we have x^2 or y^2.

Example:

$$4x - 5y = 4$$

$$6x + 2y = 25$$

If we multiply the first equation by 3 and the second equation by 2 we get $12x$ in each equation. We can then subtract the two equations from each others to obtain an equation only in y.

$$12x - 15y = 12$$

$$12x + 4y = 50$$

Subtracting these equations from each other (essentially column subtraction) gives

$$-19y = -38$$

$$y = 2$$

We must substitute back in to one of the original equations to find the value of x.

Arbitrarily choosing the first equation we get

$$4x - 5(2) = 4$$
$$4x - 10 = 4$$
$$4x = 14$$

$x = 3.5$and so the solution is $x = 3.5,\ y = 2$

Substitution can be used to solve any simultaneous equation and does not rely on the equations being set up such that one term is easily eliminated. You can substitute in an expression (for x or y) given in one equation into the other equation to end up with an equation with just one variable.

Example:

$$x + y = 11$$

$$xy = 30$$

Rewriting the first equation, we get $x = 11 - y$

Substituting this in for x in the second equation gives $(11 - y)y = 30$

which is now an equation we can solve. $11y - y^2 = 30$

$$y^2 - 11y + 30 = 0$$

$$(y - 5)(y - 6) = 0$$

$$y = 5 \text{ or } 6$$

Remembering that a solution contains both x and y terms, we substitute both values back in to one of the equations to obtain the x terms. Note we will have two solutions here.

$$x = 11 - (5) \text{ or } x = 11 - (6)$$

$$x = 6 \text{ or } 5$$

We must be careful how we give our solutions, ensuring we pair the correct x value with the correct y value. We could give the solution in two forms:

$$x = 6 \text{ and } y = 5$$

$x = 5$ and $y = 6$ or we could write the solutions as coordinate pairs

$$(6, 5) \text{ or } (5, 6)$$

Questions:

21. Solve each pair of simultaneous equations

(a) $y = 3x$, $y = 2x + 1$ (b) $y = x - 6$, $y = \frac{1}{2}x - 4$ (c) $y = 2x + 6$, $y = 3 - 4x$

(d) $x + y - 3 = 0$, $x + 2y + 1 = 0$ (e) $x + 2y + 11 = 0$, $2x - 3y + 1 = 0$ (f) $3x + 3y + 4 = 0$, $5x - 2y - 5 = 0$

22. Find the coordinates of intersection of the given straight lines and curve in each case.

(a) $y = x + 2$, $y = x^2 - 4$ (b) $y = 4x + 11$, $y = x^2 + 3x - 1$ (c) $y = 2x - 1$, $y = 2x^2 + 3x - 7$

23. Solve each pair of simultaneous equations

(a) $x^2 - y + 3 = 0$, $x - y + 5 = 0$ (b) $2x^2 - y - 8x = 0$, $x + y + 3 = 0$ (c) $x^2 + y^2 = 25$, $2x - y = 5$

(d) $x^2 + 2xy + 15 = 0$, $2x - y + 10 = 0$ (e) $x^2 - 2xy - y^2 = 7$, $x + y = 1$ (f) $3x^2 - x - y^2 = 0$, $x + y - 1 = 0$

(g) $2x^2 + xy + y^2 = 22$, $x + y = 4$ (h) $x^2 - 4y - y^2 = 0$, $x - 2y = 0$ (i) $x^2 + xy = 4$, $3x + 2y = 6$

(j) $2x^2 + y - y^2 = 8$, $2x - y = 3$ (k) $x^2 - xy + y^2 = 13$, $2x - y = 7$ (l) $x^2 - 5x + y^2 = 0$, $3x + y = 5$

(m) $3x^2 - xy + y^2 = 36$, $x - 2y = 10$ (n) $2x^2 + x - 4y = 6$, $3x - 2y = 4$ (o) $x^2 + x + 2y^2 = 52$, $x - 3y + 17 = 0$

24. Solve each pair of simultaneous equations

(a) $x - \frac{1}{y} - 4y = 0$, $x - 6y - 1 = 0$ (b) $xy = 6$, $x - y = 5$ (c) $\frac{3}{x} - 2y + 4 = 0$, $4x + y - 7 = 0$

4.3 Sketching curves

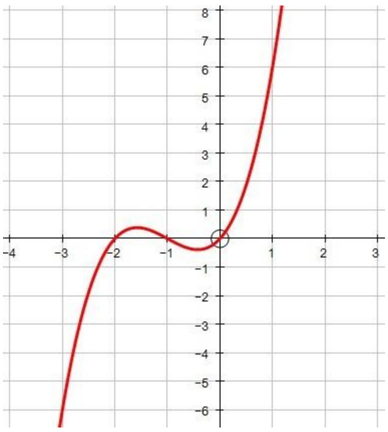
Sketching curves is often a weakness of Mathematics students, but a crucial skill for answering many questions and simplifying others. At interview for Mathematics or Mathematics-related degree courses, students often report back that they were asked to sketch a graph which they would not have prior knowledge of ($y = x^2 \sin x$, for example). It is important that throughout the course you develop your graph sketching skills, but some graphs you should already know and these are covered here.

Remember that for a sketch you always need to know the shape of the graph and where it crosses the axes (as with quadratic graphs which we covered earlier).

4.3.1 Sketching curves of cubic functions

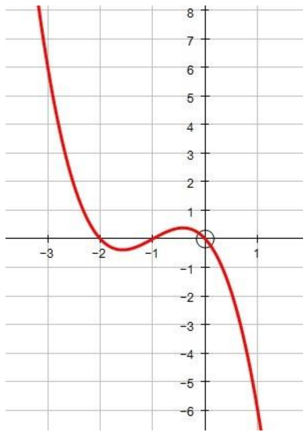
A cubic function with a positive x^3 term has the shape below.

This is the graph of $y = x(x + 1)(x + 2)$, which when expanded is the graph of $y = x^3 + 3x^2 + 2x$.



A cubic function with a negative x^3 term has the shape below.

This is the graph of $y = -x(x + 1)(x + 2)$, which when expanded is the graph of $y = -x^3 - 3x^2 - 2x$.

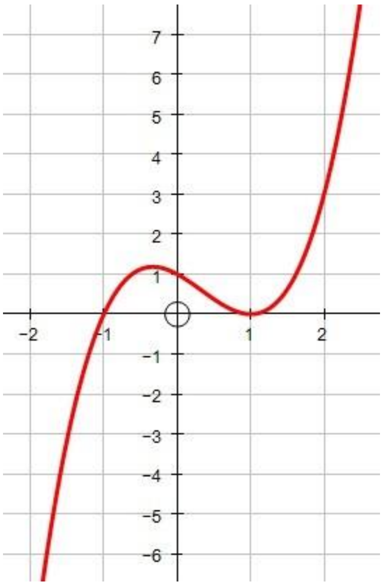


To find out where the graph crosses the axes, you follow the same procedure as with quadratics.

To find out where it crosses the y-axis you need to know what y is when $x = 0$, which is easy to find by substituting $x = 0$ into your equation.

To find out where it crosses the x-axis you need to know what x is when $y = 0$. Initially, the cubic will already be factorised or will be easily factorisable (you could factorise out a factor of x initially and then factorise the remaining quadratic) and so it becomes a case of solving the individual factors equal to 0.

Example:



When $x = 0$ (so on the y-axis),

$y = (-1)^2(1) = 1.$

When $y = 0$ (so on the x-axis),

$0 = (x-1)^2(x+1)$ so $x = 1$ (twice) or $x = -1$

The repeated root at $x = 1$ *means that the graph doesn't cross through the axis there, it only touches it, or lies tangent to it.*

Questions:

25. Sketch each graph, showing the coordinates of any points of intersection with the coordinate axes

(a) $y = (x+1)(x-1)(x-3)$ (b) $y = 2x(x-1)(x-5)$

(c) $y = -(x+2)(x+1)(x-2)$ (d) $y = x^2(x-4)$

(e) $y = 3x(2+x)(1-x)$ (f) $y = (x+2)(x-1)^2$

26. (a) Factorise fully $x^3 + 6x^2 + 9x$

(b) Hence, sketch the curve $y = x^3 + 6x^2 + 9x$, showing the coordinates of any points where the curve meets the coordinate axes.

27. Given that the constants p and q are such that $p > q > 0$, sketch each of the following graphs showing the coordinates of any points of intersection with the coordinate axes.

(a) $y = (x-p)(x-q)^2$ (b) $y = (x-p)(x^2-q^2)$

4.3.1 Sketching the reciprocal function

As with the previous two functions we have sketched, the reciprocal function looks different depending on the sign.

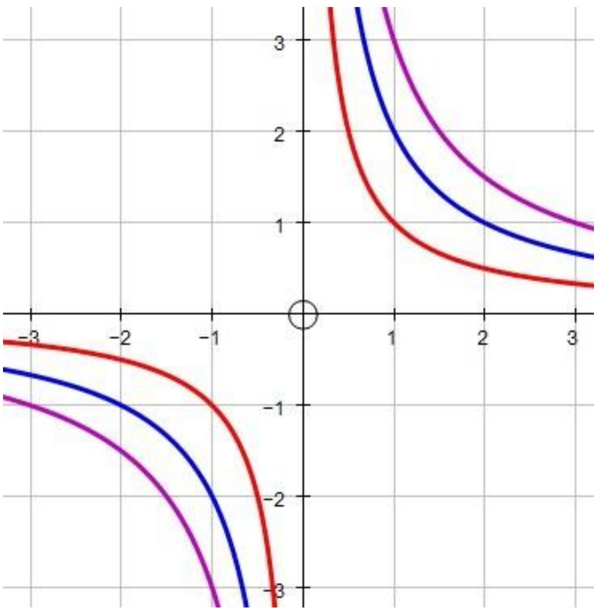
If you have $y = \frac{k}{x}$ for a positive constant k the graph looks like the one on the left.

If you have a negative constant k then the graph looks like the one on the right.

Positive Reciprocal:

This is the graph of $y = \frac{1}{x}$, $y = \frac{2}{x}$ and $y = \frac{3}{x}$

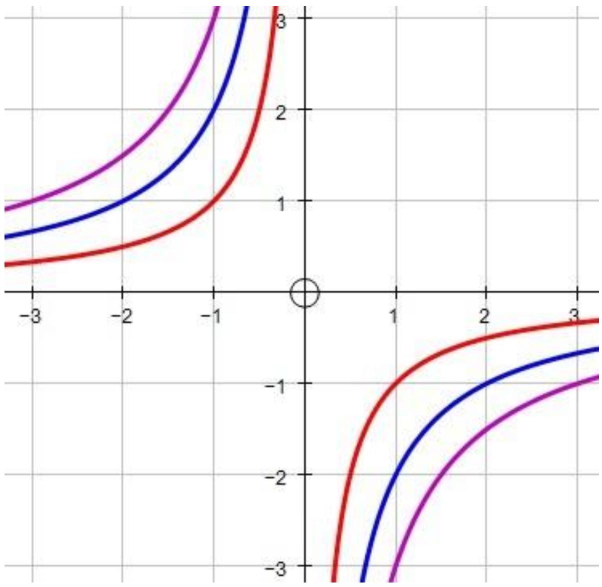
The closest graph to the axes is $y = \frac{1}{x}$



Negative Reciprocal:

This is the graph of $y = -\frac{1}{x}$, $y = -\frac{2}{x}$ and $y = -\frac{3}{x}$

The closest graph to the axes is $y = -\frac{1}{x}$



Chapter 5
Practice examination style paper

1. Expand and simplify

(a) $(2x+3)(2x-1)$ (b) $(3x-2)^2$ (c) $5x(4-x)-3(4x-8)$

2. Factorise

(a) $x^2 - 5x$ (b) $4a^2 - 81$ (c) $2x^2 + 5x - 3$ (d) $6y^2 - 13y + 5$

3. Solve the following equations

(a) $a^2 + 4a = 12$ (b) $7(6x-7) = 9x^2$ (c) $\frac{2}{y+1} + 1 = 2y$

4. Complete the square

(a) $x^2 - 6x + 11$ (b) $x^2 + 4x - 4$ (c) $2x^2 - 8x + 5$

5. Write each of these as a single power of x and/or y

(a) $(xy^2)^3$ (b) $\frac{1}{x^5}$ (c) $\frac{y^6}{y^{-2}}$ (d) $\frac{3}{x^2} \times \frac{4x^5}{3}$ (e) $\left(\frac{\sqrt{y}}{5}\right)^2$ (f) $\frac{(4\sqrt{x})^3}{16x}$

6. Solve

(a) $2^{x-1} = 16$ (b) $2(3^y - 10) = 34$ (c) $x^{\frac{3}{2}} = 64$ (d) $2^{x^2+2} = 8^x$

7. Simplify

(a) $5\sqrt{3} + \sqrt{27}$ (b) $\frac{\sqrt{5}+\sqrt{20}}{\sqrt{5}}$ (c) $\sqrt{18} \times \sqrt{50}$ (d) $\sqrt{12} - \frac{5}{\sqrt{3}}$

8. Solve $x\sqrt{12} + 9 = x\sqrt{3}$ giving your answer in the form $k\sqrt{3}$, where k is an integer.

9. Solve the simultaneous equations $x = y - 3$ and $3x^2 - 2xy + y^2 - 17 = 0$

10. Sketch the following graphs.

(a) $y = x^2 - 5x + 6$ (b) $y = -\frac{2}{x}$ (c) $y = x^3 - 2x^2 - 15x$

Chapter 6

Solutions

6.1 Solutions to Exercises

1 (a) a^7 (b) x^3 (c) b^2 (d) a (e) x^9
(f) x^{18} (g) a^4 (h) a^{12} (i) x^6 (j) b^6
(k) b^6 (l) a^2 (m) x^5 (n) 1 (o) x

2 (a) $x+x^2$ (b) $4x+2$ (c) $2x^2-2x$ (d) $8x+4x^2$
(e) $15x-10x^2$ (f) x^2+x^3 (g) x^2+3x+2 (h) x^2-1
(i) x^2+x-2 (j) x^2-5x+6 (k) $2a^2+3a+1$ (l) x^2-y^2
(m) $acx^2+(bc-ad)x-bd$ (n) $x+2x+1$

3 (a) $7x-3y$ (b) $-p$ (c) $2x+2$ (d) $2a+3b$
(e) $3m+11n$ (f) $2x-2y$ (g) $6ab-3ac-2a$
(h) m^2-n^2 (i) 0 (j) $4x-2y+15z$

4 (a) $x^2(x+1)$ (b) $x^2(2-x)$ (c) $2x^2(2x-1)$ (d) $4x^2(2x+1)$
(e) $4x^2(4-9x)$ (f) $2x^2(2x+11)$ (g) $2x^2(8-3x)$ (h) $7x^2(2x+3)$
(i) $7x^2(4x-7)$

5 (a) x^2-25 (b) $(x+5)(x-5)$ (c)(i) $(x+7)(x-7)$
(c)(ii) $(x+8)(x-8)$ (c)(iii) $(x+10)(x-10)$ (c)(iv) $(x+a)(x-a)$
(c)(v) $(x+2b)(x-2b)$

6 (a) $(x+3)(x+4)$ (b) $(x+1)(x+7)$ (c) $(x+2)(x+9)$
(d) $(x+3)(x+9)$ (e) $(x+7)(x+10)$ (f) $(x+2)(x+4)$
(g) $(x+2)(x+14)$ (h) $(x+7)(x+11)$ (i) $(x+7)(x+9)$

7 (a) $(x+2)(x-1)$ (b) $(x+5)(x-4)$ (c) $(x+3)(x-4)$
(d) $(x-4)(x-9)$ (e) $(x-2)(x-8)$ (f) $(x+7)(x-6)$
(g) $(x+15)(x-2)$ (h) $(x-8)(x-9)$ (i) $(x+9)(x-11)$

8 (a) $(2x+1)(x+1)$ (b) $(3p+1)(p+2)$ (c) $(2y-3)(y-1)$

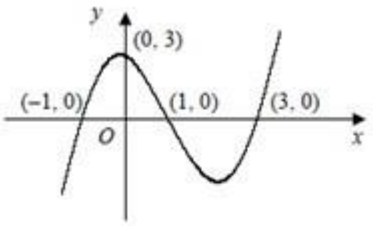
	(d)	$(2+m)(1-m)$	(e)	$(3r+1)(r-1)$	(f)	$(5+y)(1-4y)$
	(g)	$(3a-1)(a-4)$	(h)	$(5x+2)(x-2)$	(i)	$(2x+1)(2x+3)$
	(j)	$(3s-1)^2$	(k)	$(2m+5)(2m-5)$	(l)	$(2+3y)(1-2y)$
	(m)	$(4u+1)(u+4)$	(n)	$(3p+4)(2p-1)$	(o)	$(8x+3)(x+2)$
9	(a)	$8\sqrt{2}$	(b)	$\sqrt{3}$	(c)	$10\sqrt{2}$
	(d)	$2\sqrt{10}$	(e)	$5\sqrt{5}$	(f)	$-\sqrt{6}$
10	(a)	$3+2\sqrt{3}$	(b)	$2+\sqrt{3}$	(c)	$5+3\sqrt{3}$
	(d)	$10+9\sqrt{3}$	(e)	$43-23\sqrt{3}$	(f)	$-43+\sqrt{3}$
11	(a)	$13+5\sqrt{5}$	(b)	$7\sqrt{2}-11$	(c)	$37+12\sqrt{7}$
	(d)	$7+13\sqrt{2}$	(e)	$1+\sqrt{10}$	(f)	$8-5\sqrt{2}$
12	(a)	$\frac{1}{5}\sqrt{5}$	(b)	$\frac{2}{3}\sqrt{3}$	(c)	$\frac{1}{4}\sqrt{2}$
	(d)	$2\sqrt{7}$	(e)	$\sqrt{6}$	(f)	$\frac{1}{3}\sqrt{3}$
	(g)	$\frac{1}{21}\sqrt{7}$	(h)	$\sqrt{2}$	(i)	$\frac{1}{20}\sqrt{5}$
	(j)	$\frac{1}{12}\sqrt{6}$	(k)	$\frac{4}{9}\sqrt{10}$	(l)	$\frac{5}{6}\sqrt{21}$
13	(a)	$x=-7$ or $x=5$	(b)	$x=18$ or $x=-3$	(c)	$x=10$ or $x=-9$
	(d)	$x=-9$ or $x=-6$	(e)	$x=-3$ or $x=-17$	(f)	$x=4$ or $x=8$
	(g)	$x=11$ or $x=13$	(h)	$x=12$ or $x=5$	(i)	$x=22$ or $x=-8$
	(j)	$x=19$ or $x=7$	(k)	$x=-11$ or $x=4$	(l)	$x=-15$ or $x=-\frac{1}{2}$
	(m)	$x=3$ or $x=-\frac{1}{2}$	(n)	$x=\frac{9}{2}$ or $x=-1$	(o)	$x=3$ or $x=-\frac{1}{2}$
14	(a)	$x=4$ or $x=-4$	(b)	$x=7$ or $x=-7$	(c)	$x=\frac{9}{2}$ or $x=-\frac{9}{2}$
	(d)	$x=\frac{8}{3}$ or $x=-\frac{8}{3}$				
15	(a)	$q=3$	(b)	$x=9$	(c)	$y=11$
	(d)	$x=\frac{2}{3}$	(e)	$y=\frac{5}{2}$		
16	(a)	$x=\pm 5$	(b)	$a=\pm 6$	(c)	$y=\pm\frac{7}{2}$
	(d)	$b=\pm 4$	(e)	$a=\pm 8$	(f)	$x=\pm\frac{2}{9}$
	(g)	$y=\pm\frac{3}{2}$	(h)	$x=\pm 4$	(i)	$p=\pm 3$

	(j)	$p = \pm 2$	(k)	$p = \pm\frac{7}{5}$	(l)	$b = \pm 2$
17	(a)	$y = 8$ or $y = -7$	(b)	$w = \frac{3}{4}$ or $w = \frac{1}{3}$	(c)	$y = -\frac{4}{3}$ or $y = -\frac{1}{2}$
	(d)	$c = -1$ or $c = 2$	(e)	$q = 2$ or $q = -4$	(f)	$d = 1$ or $d = -3$
	(g)	$x = 12$ or $x = 7$	(h)	$y = -6$ or $y = \frac{3}{5}$	(i)	$p = 0$ or $p = -4$
	(j)	$x = \frac{1}{4}$ or $x = -\frac{3}{2}$	(k)	$x =$ or $x = 2$	(l)	$x = \frac{5}{2}$ or $x = -4$
	(m)	$y =$ or $y = 3$	(n)	$y = -\frac{1}{2}$ or $y = -3$	(o)	$a = 1$ or $a = -3$
	(p)	$x = 3$ or $x = \frac{7}{3}$				
18	(a)	$(x+1)^2+3$	(b)	$(x-1)^2+3$	(c)	$(x-2)^2-3$
	(d)	$(x+3)^2-9$	(e)	$(x+2)^2+4$	(f)	$(x-4)^2-21$
	(g)	$(x+6)^2-6$	(h)	$(x-5)^2$	(i)	$(x+3)^2-18$
	(j)	$(x-2)^2+14$	(k)	$\left(x+\frac{3}{2}\right)^2+\frac{3}{4}$	(l)	$\left(x+\frac{1}{2}\right)^2-\frac{5}{4}$
	(m)	$(x-9)^2+19$	(n)	$\left(x-\frac{1}{2}\right)^2-\frac{3}{4}$	(o)	$\left(x+\frac{9}{2}\right)^2-\frac{1}{4}$
	(p)	$\left(x-\frac{7}{2}\right)^2-\frac{57}{4}$	(q)	$\left(x-\frac{3}{2}\right)^2+\frac{11}{4}$	(r)	$\left(x-\frac{11}{2}\right)^2+\frac{27}{4}$
	(s)	$\left(x+\frac{1}{3}\right)^2+\frac{8}{9}$	(t)	$\left(x-\frac{1}{4}\right)^2-\frac{5}{16}$		
19	(a)	$2(x+1)^2+1$	(b)	$2(x-2)^2-15$	(c)	$3(x-1)^2$
	(d)	$4(x+3)^2-25$	(e)	$-(x+1)^2-4$	(f)	$-(x-5)^2+26$
	(g)	$2\left(x+\frac{1}{2}\right)^2-\frac{3}{2}$	(h)	$3\left(x-\frac{3}{2}\right)^2-\frac{7}{4}$	(i)	$3(x-4)^2$
	(j)	$3\left(x-\frac{5}{2}\right)^2-\frac{75}{4}$	(k)	$5(x+4)^2-10$	(l)	$2\left(x+\frac{5}{4}\right)^2-\frac{9}{8}$
	(m)	$4\left(x+\frac{3}{4}\right)^2+\frac{37}{4}$	(n)	$-2(x-1)^2+1$	(o)	$-3\left(x+\frac{1}{3}\right)^2+\frac{13}{3}$
	(p)	$\frac{1}{3}\left(x+\frac{3}{4}\right)^2-\frac{7}{16}$				
20	(a)	$y = 2 \pm \sqrt{2}$	(b)	$p = -1 \pm \sqrt{3}$	(c)	$x = 3 \pm \sqrt{5}$
	(d)	$r = -5 \pm 3\sqrt{2}$	(e)	$x = 1 \pm 2\sqrt{3}$	(f)	$a = 7 \pm 3\sqrt{6}$
	(g)	$m = \frac{1}{2}(3 \pm \sqrt{5})$	(h)	$y = \frac{1}{2}(7 \pm \sqrt{13})$	(i)	$u = -11$ or 4
	(j)	$y = 1 \pm \frac{1}{2}\sqrt{2}$	(k)	$p = 3 \pm \frac{2}{3}\sqrt{3}$	(l)	$x = -3 \pm \frac{3}{2}\sqrt{6}$

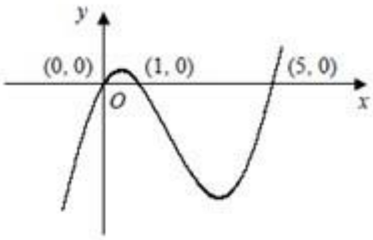
	(m)	$m = \frac{1}{2}(1 \pm \sqrt{5})$	(n)	$x = \frac{7}{2}$	(o)	$t = \frac{1}{6}(-1 \pm \sqrt{13})$
	(p)	$a = \frac{1}{4}(7 \pm \sqrt{17})$				
21	(a)	$x = 1$ and $y = 3$	(b)	$x = 4$ and $y = -2$	(c)	$x = -\frac{1}{2}$ and $y = 5$
	(d)	$x = 7$ and $y = -4$	(e)	$x = -5$ and $y = -3$	(f)	$x = \frac{1}{3}$ and $y = -\frac{5}{3}$
22	(a)	$(-2, 0)$ and $(3, 5)$	(b)	$(-3, -1)$ and $(4, 27)$	(c)	$(-2, -5)$ and $(\frac{3}{2}, 2)$
23	(a)	$x = -1, y = 4$ or $x = 2, y = 7$	(b)	$x = \frac{1}{2}, y = -\frac{7}{2}$ or $x = 3, y = -6$	(c)	$x = 0, y = -5$ or $x = 4, y = 3$
	(d)	$x = -3, y - 4$ or $x = -1, y = 8$	(e)	$x = -2, y = 3$ or $x = 2, y = -1$	(f)	$x = -1, y = 2$ or $x = \frac{1}{2}, y = \frac{1}{2}$
	(g)	$x = -1, y = 5$ or $x = 4, y = -3$	(h)	$x = 0, y = 0$ or $x = \frac{8}{3}, y = \frac{4}{3}$	(i)	$x = 2, y = 0$ or $x = \frac{5}{2}, y = -\frac{5}{2}$
	(j)	$x = 2, y = 1$ or $x = 5, y = 7$	(k)	$x = 3, y = -1$ or $x = 4, y = 1$	(l)	$x = 1, y = 2$ or $x = \frac{5}{2}, y = -\frac{5}{2}$
	(m)	$x = -2, y = -6$ or $x = 2, y = -4$	(n)	$x = \frac{1}{2}, y = -\frac{5}{4}$ or $x = 2, y = 1$	(o)	$x = -5, y = 4$ or $x = -2, y = 5$
24	(a)	$x = -5, y = -1$ $x = 4, y = \frac{1}{2}$	(b)	$x = -1, y = -6$ $x = 6, y = 1$	(c)	$x = \frac{1}{2}, y = 5$ $x = \frac{3}{4}, y = 4$

25

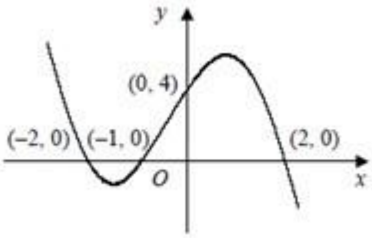
(a)



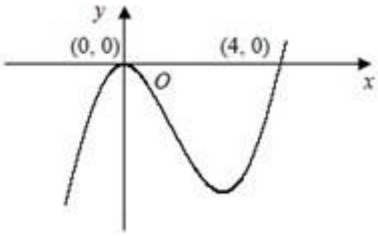
(b)



(c)



(d)



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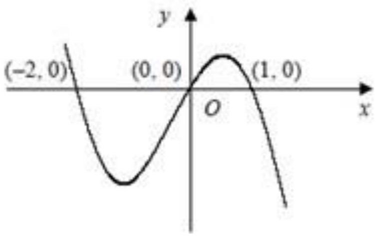
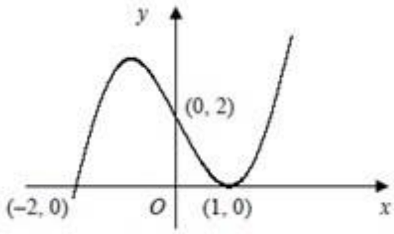
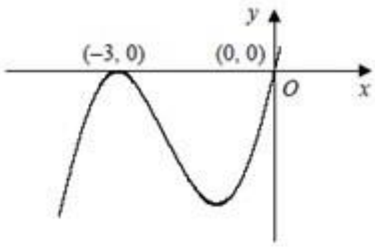


(f

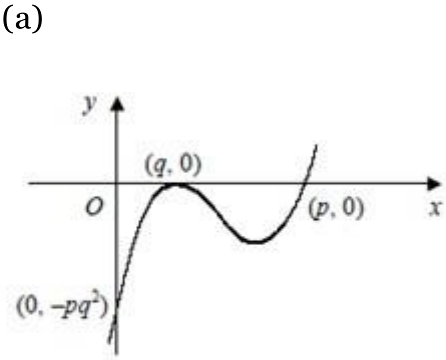


26 (a) $x(x + 3)^2$

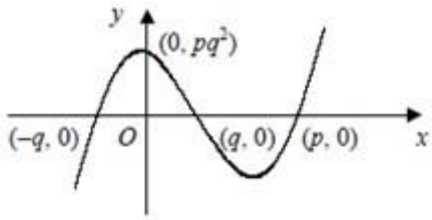
(b)



27 (a)



(b) $y = (x - p)(x + q)(x - q)$



6.2 Solutions to Practice examination style paper

1 (a) $4x^2 + 4x - 3$ (b) $9x^2 - 12x + 4$ (c) $8x - 5x^2 + 24$

2 (a) $x(x - 5)$ (b) $(2a + 9)(2a - 9)$ (c) $(2x - 1)(x + 3)$ (d) $(3y + 1)(2y - 5)$

3 (a) $a = 2$ or $a = -6$ (b) $x = \frac{7}{3}$ (c) $y = -\frac{3}{2}$ or $y = 1$

4 (a) $(x - 3)^2 + 2$ (b) $(x + 2)^2 - 20$ (c) $2(x - 2)^2 - 3$

5 (a) x^2y^6 (b) x^{-5} (c) y^8 (d) $4x^3$ (e) $\frac{y}{25}$ (f) $4x^{\frac{1}{2}}$

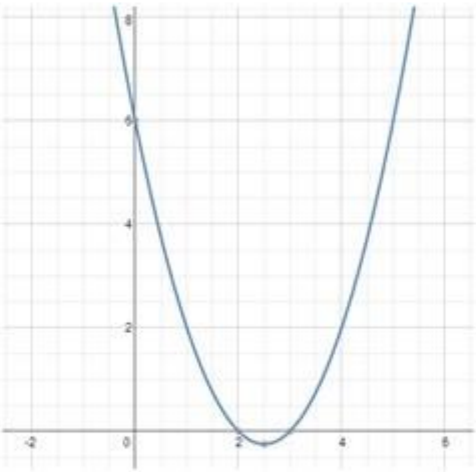
6 (a) x=5 (b) y=3 (c) x=16 (d) x=2 or x=1

7 (a) $8\sqrt{3}$ (b) 3 (c) 30 (d) $\frac{1}{\sqrt{3}}$

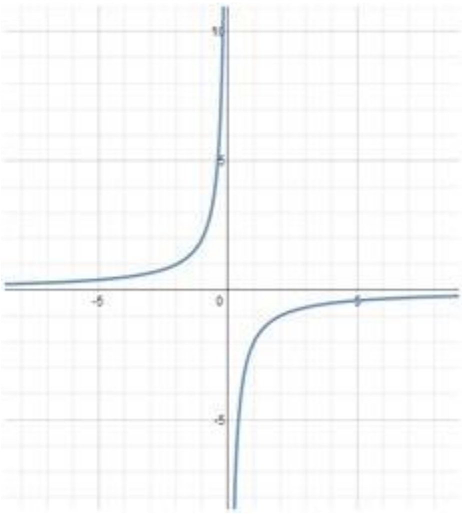
8 $x = -3\sqrt{3}$

9 $x = -2, y = 1$ and $x = 2, y = 5$

10 (a)



(b)



(c)