*1–7 *The Uses of Dimensional Analysis*

53. (II) We have seen in the text that the period of a simple pendulum is independent of the mass of the pendulum bob. Further, we have seen that the dimensional relation between the period, τ, the length, ℓ, of the pendulum, and the acceleration of gravity, g, takes the form

$$[\tau] = [\ell^r][g^s].$$

Use the fact that the dimension of τ is [T], that of ℓ is [L], and that of g is [L/T^2] to show that

$$\tau = \sqrt{\frac{\ell}{g}}.$$

54. (II) In quantum mechanics, the fundamental constant called Planck's constant, h, has dimensions of $[ML^2T^{-1}]$. Construct a quantity with the dimensions of length from h, a mass m, and c, the speed of light.

55. (II) It is known that the quantity Ke^2/hc is dimensionless (K is a numerical constant; h and c are as discussed in Problem 54). (a) What are the dimensions of e? (b) What are the dimensions of e^2/R, where R is a length?

56. (II) You are told that the speed of sound in a metal depends only on the density ρ ($[ML^{-3}]$) and on the bulk modulus of the metal, B, which has dimensions $[ML^{-1}\,T^{-2}]$. Express the sound speed in terms of ρ and B.

72. (II) A mouse is 10 cm in length, whereas an elephant is 4 m in length. The amount of food an animal must eat is proportional to its heat loss, and the heat loss is proportional to its surface area. Compare the percentage of body weight that a mouse and an elephant must eat each day. Ignore the detailed differences in shape between an elephant and a mouse.

81. (III) A stretched wire has three physical attributes: the density λ, or mass per unit length; the total length ℓ; and the tension τ. The latter is related to how hard the wire is being pulled to keep it stretched, and has dimensions of $[MLT^{-2}]$. Show by dimensional analysis that if the time t_0 of one back-and-forth vibration of the wire in a direction perpendicular to its length depends only on these three quantities, then t_0 has the form $t_0 = (\text{a constant})\ \ell \sqrt{\lambda/\tau}$.

66. (II) The gasoline usage rate required to propel an automobile is very roughly proportional to the mass of the automobile. Assuming that the proportions and types of materials of an automobile do not change, calculate the percentage gasoline savings that would be realized if cars were reduced by 12 percent in all their dimensions.

67. In aquatic animals, the energy E available for motion is proportional to the mass of the animal, and the friction F with their skin is proportional to the surface area. All such animals have the same density, very close to that of water. If the maximum speed v such an animal can reach varies as $\sqrt{E/F}$, show that v is also proportional to $\sqrt{L}$, where L is some length characterizing the animal's size.